

Riemann Sums - Midpoint

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration, Applications of Differentiation

Subtopics: Average Value of a Function, Riemann Sums – Midpoint, Interpreting Meaning in Applied Contexts, Mean Value Theorem

Paper: Part A-Calc / Series: 2003-Form-B / Difficulty: Somewhat Challenging / Question Number: 3

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.
- (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.
- (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.
- (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.
- (d) Explain why there must be at least one value x , for $0 < x < 360$, such that $B''(x) = 0$.

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Question 2

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Applications of Integration

Subtopics: Riemann Sums – Midpoint, Interpreting Meaning in Applied Contexts, Mean Value Theorem, Kinematics (Displacement, Velocity, and Acceleration), Average Value of a Function

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Medium / Question Number: 3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

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Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Rates of Change (Average), Interpreting Meaning in Applied Contexts, Riemann Sums – Midpoint

Paper: Part B-Non-Calc / Series: 2006 / Difficulty: Medium / Question Number: 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

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Question 4

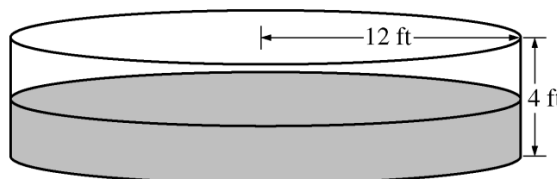
Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Riemann Sums – Midpoint, Total Amount, Modelling Situations, Rates of Change (Instantaneous), Related Rates, Accumulation of Change

Paper: Part A-Calc / Series: 2010-Form-B / Difficulty: Easy / Question Number: 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)
- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
 - Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
 - Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
 - Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.
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Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Rates of Change (Average), Mean Value Theorem, Riemann Sums – Midpoint, Interpreting Meaning in Applied Contexts, Rates of Change (Instantaneous)

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.
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